

# Self-cooling of a movable mirror to the ground state using radiation pressure

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We show that one can cool a micro-mechanical oscillator to its quantum ground state using radiation pressure in an appropriately detuned cavity (self-cooling). From a simple theory based on Heisenberg-Langevin equations we find that optimal self-cooling occurs in the good cavity regime, when the cavity bandwidth is smaller than the mechanical frequency, but still larger than the effective mechanical damping. In this case the intracavity field and the vibrational mechanical mode coherently exchange their fluctuations. We also present dynamical calculations which show how to access the mirror final temperature from the fluctuations of the field reflected by the cavity.

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Cooling of mechanical resonators both at the micro- and at the macro-level, is an important technical challenge in various fields of physics, such as ultra-high precision measurements [1], and detection of gravitational waves [2]. It is also a prerequisite for any possible use of optomechanical systems for quantum information processing [3, 4]. Active noise control techniques have been proposed to reduce their thermal noise and bring the oscillator motion to its ground state [5]. Recently, various experiments have demonstrated significant cooling of the vibrational mode coupled to an optical cavity [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. The experiments of Refs. [8, 10, 12, 13] in particular have adopted the so-called *back-action* [16], or *self-cooling* scheme, in which the radiation pressure of an appropriately detuned cavity interacting with the mechanical oscillator, is used. In all these experiments, however, the resulting equilibrium state of the oscillator is classical, because the new mean excitation number is still much larger than one. Therefore it is important to establish the fundamental limits of self-cooling and if it is able to cool a mechanical degree of freedom down to its quantum ground state. We will describe the system dynamics in terms of quantum Langevin equations (QLE) for the Heisenberg operators of the system and we will show that ground state self-cooling is possible in the good cavity regime, i.e. when the mechanical resonance frequency is larger than the cavity bandwidth, and provided the cavity detuning and bandwidth are appropriately chosen so as to maximize the scattering of noisy photons into the cavity mode.

We consider a driven optical cavity coupled by radiation pressure to a micromechanical oscillator. The typical experimental configuration is a Fabry-Perot cavity with one mirror much lighter than the other (see e.g. [7, 8, 9, 10, 11, 15]), but our treatment applies to other optomechanical systems, such as the silica toroidal micro-cavity of Refs. [12, 17]. Radiation pressure typically excites several mechanical degrees of freedom of the system

with different resonant frequencies. However, we consider the detection in a narrow frequency bandwidth, containing a single mechanical resonance peak. In this case the oscillator dynamics can be well approximated by that of a single harmonic oscillator with dimensionless position  $q(t)$ , and momentum  $p(t)$ , ( $[q, p] = 2i$ ), frequency  $\Omega_m$ , mass  $M$ , and damping  $\Gamma$ . The coupled dynamics of the mirror with the intracavity field mode  $a(t)$  in the frame rotating at the frequency of the driving laser  $\omega_L$ , is described by the QLE

$$\dot{q} = \Omega_m p \quad (1)$$

$$\dot{p} = -\Omega_m q - \Gamma p + G\sqrt{2}a^\dagger a + \xi \quad (2)$$

$$\dot{a} = -(\kappa + i\Delta_c)a + iGaq/\sqrt{2} + \sqrt{2\kappa}a^{\text{in}}, \quad (3)$$

where  $G = (\omega_c/L)\sqrt{\hbar/M\Omega_m}$  is the opto-mechanical coupling constant,  $\omega_c$  and  $L$  being the cavity resonance frequency and length, respectively.  $a^{\text{in}}(t)$  is the input field, satisfying  $\langle a^{\text{in}}(t)a^{\text{in}\dagger}(t') \rangle - |\bar{a}^{\text{in}}|^2 = \delta(t - t')$ , with  $\bar{a}^{\text{in}}$  the incident mean field.  $\kappa$  is the damping rate of the cavity mode,  $\Delta_c = \omega_c - \omega_L$  the cavity detuning and  $\xi$  the noise operator accounting for the mirror Brownian motion at thermal equilibrium at temperature  $T$  [18].

*Steady state analysis* - In steady state the mean intracavity field  $\bar{a}$  is given by  $\bar{a} = \sqrt{2\kappa}\bar{a}^{\text{in}}/(\kappa + i\Delta)$ , where  $\Delta$  is the mean detuning of the cavity, given by  $\Delta = \Delta_c - \Delta_{nl} = \Delta_c - G^2|\bar{a}|^2/\Omega_m$ . These two coupled equations give a third-order relation between  $\bar{a}$  and  $\Delta$  which leads to the well-known bistable behavior of a cavity with a movable mirror [19]. The stability condition of the system can be written as  $\kappa^2 + \Delta^2 + 2\Delta\Delta_{nl} > 0$ .

If we then consider the linearized fluctuations of the various operators around the steady state, we get linearized QLE which can be solved by moving to the frequency domain. The Fourier transform of the position fluctuations can then be simply expressed as the sum of

a thermal noise term and a radiation pressure term

$$q[\Omega] = \tilde{\chi}[\Omega] (F_R[\Omega] + F_T[\Omega]), \quad (4)$$

the response to the noise terms being given by an effective susceptibility

$$\tilde{\chi}[\Omega]^{-1} = \chi[\Omega]^{-1} - M\Omega_m^2 \frac{2\Delta_{nl}\Delta}{\kappa^2 D[\Omega]} \quad (5)$$

with  $\chi[\Omega] = M\Omega_m^2 [1 - \Omega^2/\Omega_m^2 - i\Gamma\Omega/\Omega_m^2]$  and  $D[\Omega] = (1 - i\Omega/\kappa)^2 + (\Delta/\kappa)^2$ . The thermal noise force is simply  $F_T[\Omega] = M\Omega_m\xi[\Omega]$  and the radiation pressure force [20]

$$F_R[\Omega] = \frac{\sqrt{2}M\Omega_m^{\frac{3}{2}}\Delta_{nl}^{\frac{1}{2}}}{\kappa^{\frac{3}{2}}D[\Omega]} [(\kappa - i\Omega)x^{\text{in}}[\Omega] + \Delta y^{\text{in}}[\Omega]]$$

with  $x^{\text{in}} = a^{\text{in}} + a^{\text{in}\dagger}$  and  $y^{\text{in}} = i(a^{\text{in}\dagger} - a^{\text{in}})$ . An exact expression for the mirror variances is then obtained by integrating the contributions of these two forces to the noise spectrum ( $\omega = \Omega/\Omega_m$ )

$$\Delta q^2 = \int \frac{d\omega}{2\pi} S_q(\omega), \quad \Delta p^2 = \int \frac{d\omega}{2\pi} \omega^2 S_q(\omega) \quad (6)$$

where we have used the fact that  $p = \dot{q}/\Omega_m$  and where the noise spectrum is given by

$$S_q(\omega) = \left[ \frac{2\omega \coth\left(\frac{\hbar\Omega_m\omega}{2k_B T}\right)}{Q} + 4\varphi_{nl} \frac{1 + \varphi^2 + b^2\omega^2}{(1 - b^2 + \varphi^2)^2 + 4b^2\omega^2} \right] \frac{|(1 - ib\omega)^2 + \varphi^2|^2}{|[(1 - ib\omega)^2 + \varphi^2][1 - \omega^2 - i\omega/Q] - 2\varphi\varphi_{nl}|^2}, \quad (7)$$

with  $b = \Omega_m/\kappa$ ,  $\varphi = \Delta/\kappa$ ,  $\varphi_{nl} = \Delta_{nl}/\kappa$  and  $Q = \Omega_m/\Gamma$ . These two exact expressions for the variances coincide with Eq. 5 of Ref. [21]. Cooling to the ground state implies reaching  $\Delta q^2 = 1$  simultaneously with  $\Delta p^2 = 1$ . Fig. 1 shows both variances as a function of  $b$ , for a cavity detuning equal to one bandwidth ( $\varphi = b$ ), which shows that, starting from a mean thermal excitation number  $n_T^i = (\exp\{\hbar\Omega_m/k_B T\} - 1)^{-1}$  equal to  $10^2$ , it is possible to achieve a lower than unity final excitation number state in the *good* cavity limit:  $b > 1$ , i.e.  $\Omega_m > \kappa$ .

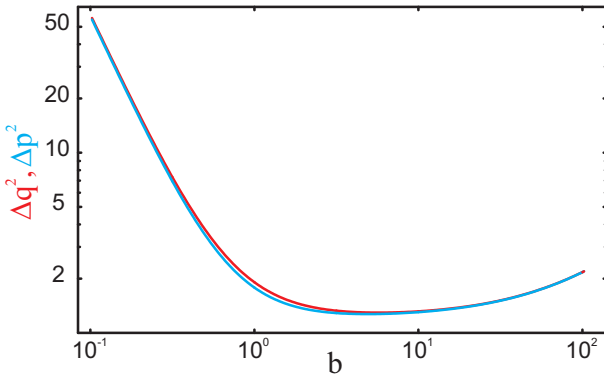


FIG. 1: Normalized variances versus  $b$ , for a detuning  $\varphi = b$ ,  $Q = 10^4$ ,  $n_T^i = 10^2$ ,  $\varphi_{nl} = 0.1$ . The final excitation number is  $n_T^f \sim 0.15$  for  $b \sim 5$ .

*Physical interpretation in the adiabatic limit* - To get more insight into the cooling mechanism we will now derive an analytical expression for the frequency integrals of (6). The latter may be considerably simpli-

fied by noting that, for high-Q cavities and under certain conditions that we will detail, the mirror response is peaked around the natural mechanical resonance frequency. One may then express the effective susceptibility as  $\tilde{\chi}[\Omega]^{-1} \simeq M\tilde{\Omega}^2[1 - \Omega^2/\tilde{\Omega}^2 - i/\tilde{Q}]$  with an effective quality factor  $\tilde{Q} = \tilde{\Omega}/\tilde{\Gamma}$  and effective resonance frequency and relaxation rate

$$\tilde{\Omega} = \Omega_m [1 - 2\varphi\varphi_{nl} \text{Re}[D[\Omega_m]^{-1}]]^{1/2} \quad (8)$$

$$\tilde{\Gamma} = \Gamma [1 + 2\varphi\varphi_{nl}Q \text{Im}[D[\Omega_m]^{-1}]] \quad (9)$$

The effect of the frequency-shift in the resonance is typically negligible, so that one may assume  $\tilde{\Omega} \simeq \Omega_m$ . The relaxation rate of the mirror is on the contrary strongly affected by the presence of the cavity field and, depending on the sign of the cavity detuning, may either be enhanced or reduced, resulting in either cooling or heating. As pointed out in [22, 23, 24], the most interesting regime for self-cooling is when the effective damping rate  $\tilde{\Gamma}$  is strongly enhanced, but still less than the cavity bandwidth and when the effective mechanical  $\tilde{Q}$  is still larger than one,  $\Gamma \ll \tilde{\Gamma} \ll \kappa$ . In this case, the effective susceptibility is still peaked around  $\omega = 1$  and we can well approximate the smoothly varying function of  $\omega$  inside the square brackets by taking its value at  $\omega = 1$ . One then gets an analytical expression for the variance

$$\Delta q^2 \simeq \frac{\Gamma}{\tilde{\Gamma}} \left[ 2n_T^i + 1 + 2\varphi_{nl}Q \frac{1 + b^2 + \varphi^2}{(1 - b^2 + \varphi^2)^2 + 4b^2} \right] \quad (10)$$

Eq. (10) is the basic equation for our analysis of the quantum limits of self-cooling, and it is possible to see that it coincides with the results of [22, 24], which were obtained with a different method. Ref. [21] instead gives

slightly more general expressions for the two variances, which, however, reduce to that of Eq. (10) when  $\Omega_m$  is large enough, i.e.,  $\Omega_m > \tilde{\Gamma}$ .

Depending on the respective frequencies involved, one can adopt different strategies to cool the mirror down to the ground state. In particular, the cavity detuning has to be well-chosen in order to optimize the self-cooling. Fig. 2 shows the variation of the variance with the cavity detuning, as calculated from the exact solution of Eq. (7) and from the harmonic approximation (10). The analytical result is in good agreement with the numerical result and we will use it as a basis for the cooling optimization discussion.

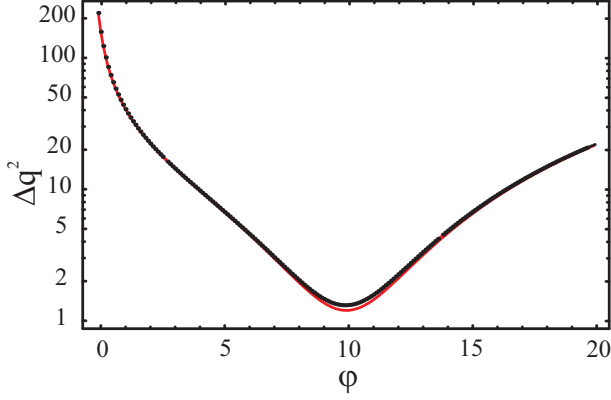


FIG. 2: Normalized variance versus cavity detuning [dashed: exact result of Eq. (7), plain: approximate result of Eq. (10)]. Parameters:  $Q = 10^4$ ,  $n_T^i = 10^2$ ,  $\varphi_{nl} = 0.1$ ,  $b = 10$ .

In order to interpret Eq. (10) we rewrite it as

$$\Delta q^2 = \frac{1 + 2n_T^i}{1 + f\varphi_{nl}Q} + \frac{f\varphi_{nl}Q}{1 + f\varphi_{nl}Q} \frac{1 + b^2 + \varphi^2}{2\varphi b}, \quad (11)$$

or, equivalently,

$$\Delta q^2 = (1 - \eta) \Delta q_T^2 + \eta \Delta q_R^2. \quad (12)$$

$\Delta q_T^2 = 1 + 2n_T^i$  and  $\Delta q_R^2 = (1 + b^2 + \varphi^2)/2\varphi b$  represent the thermal noise-induced and the cavity field-induced contributions, respectively, to the position fluctuations of the mirror.

$$\eta = \frac{f\varphi_{nl}Q}{1 + f\varphi_{nl}Q} \quad (13)$$

is the weight between those two quantities, with

$$f = \frac{4\varphi b}{(1 - b^2 + \varphi^2)^2 + 4b^2}. \quad (14)$$

The physical interpretation of Eq. (12) is that the cavity field and the mirror vibrational mode coherently exchange their fluctuations during the interaction, all the more so that the quantity  $\eta$  is close to unity. The thermal excess noise is then replaced by the cavity field fluctuations, which are those of the vacuum, hence resulting in

decreasing the effective temperature of the mirror. The quantity  $\eta$  naturally appears as a quantum state transfer efficiency between the field and the vibrational mode, whereas the product  $\varphi_{nl}Q$  can be interpreted as a coherent coupling strength, quite similarly to the *cooperativity* parameter, which appears in quantum state transfer schemes between optical fields and atomic ensembles [25, 26, 27]. Since the mirror is initially in a noisy thermal state, the cavity field acts as a thermal noise “eater” during the interaction, and allows to reach a much lower effective temperature for the mirror in steady state. This situation is also quite similar to cavity assisted Doppler-cooling of atoms [28], for which the temperature of the atoms is decreased by enhancing the scattering of photons into the cavity mode.

If one starts with a very large thermal excess noise, Eq. (11) clearly shows that the thermal noise contribution will be suppressed by a factor  $f\varphi_{nl}Q$ , which is typically large. Minimizing the mirror final temperature is then equivalent to maximizing the transfer efficiency  $\eta$ , and, for a given value of  $\varphi_{nl}Q$ , maximizing  $f$  with respect to  $\varphi$ . This gives an optimal cavity detuning equal to

$$\varphi^* = [(b^2 - 1 + 2\sqrt{1 + b^2 + b^4})/3]^{\frac{1}{2}}. \quad (15)$$

The most favorable conditions to reach the quantum ground state of the mechanical oscillator then occur for a cavity bandwidth *smaller* than the mechanical resonance. Indeed, assuming  $b \gg 1$ , one has  $\varphi^* \simeq b$ ,  $f^* \simeq 1$  and for a sufficiently high Q-factor ( $\varphi_{nl}Q \gg 2n_T^i + 1$ ), the resulting mirror fluctuations are then mostly given by the radiation pressure:

$$(\Delta q^2)^* \simeq \Delta q_R^2 \simeq 1 + \frac{1}{2b^2} \quad (16)$$

and the normalized variance tends to unity for  $b \gg 1$ . It is therefore possible to reach the mechanical ground state using self-cooling in the *good* cavity limit.

However, there exists an optimal bandwidth that minimizes the mirror final temperature. Indeed, as one decreases the cavity bandwidth (for a fixed value of the mechanical resonance frequency), the effective response time of the mirror becomes comparable to that of the intracavity field and the previous adiabatic approximation is no longer valid. One can also show that this situation is detrimental to the noise transfer between the field and the mirror [25, 26]. As mentioned previously and as can be seen in Fig. 1, the adiabatic approximation breaks down when  $\varphi_{nl}\Omega_m \gtrsim 2\kappa$ .

*Dynamical situation* - We now address the dynamical behavior of the coupled field-mirror system. Under good self-cooling conditions one expects the mirror initial thermal noise to be mainly transferred to the cavity field during the cooling phase, and hence to the field leaking out of the cavity. Numerically solving the linearized Langevin equations in time allows to compute the two-time correlation functions of  $q(t)$  and  $p(t)$ , as well as those

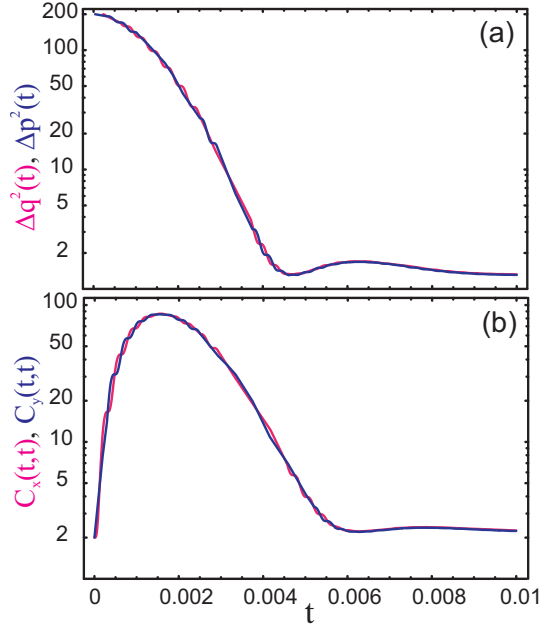


FIG. 3: (a) Temporal evolution of the mirror position and momentum normalized variances when the cooling field is on from  $t = 0$  ( $\Gamma^{-1}$  units). (b) Temporal evolution of the outgoing field variances  $C_{x,y}(t, t)$  (in  $\kappa$  units). Parameters:  $\kappa = 10^3\Gamma$ ,  $Q = 10^4$ ,  $n_T^i = 10^2$ ,  $\varphi_{nl} = 0.1$ ,  $\varphi = b = 10$ .

of the outgoing field quadratures,  $x^{\text{out}}$  and  $y^{\text{out}}$ , where  $a^{\text{out}} = \sqrt{2\kappa a} - a^{\text{in}}$ . To simplify, we assume a step-like injection of a coherent state cooling field into the cavity at  $t = 0$  and start with an occupation number  $n_T^i = 10^2$  for the mirror [29]. The resulting mirror variances at time

$t$  are represented in Fig. 3, as well as the temporal evolution of the equal-time variances of the outgoing field  $C_{x,y}(t, t)$ , obtained from exact numerical calculations. It appears clearly that, during the cooling phase, thermal noise is taken from the mirror and transferred to the cavity field. The amount of thermal noise taken from the mirror can be deduced by measuring the outgoing field fluctuations. Indeed, for  $\Omega_m = \Delta$  and in the adiabatic limit  $\Gamma \ll \tilde{\Gamma} \ll \kappa$ , it can be shown that the outgoing field correlation functions are of the form [23, 31]

$$C_{x,y}(t, t') = 2\eta\tilde{\Gamma}e^{-\tilde{\Gamma}(t+t')} + n_T^i\eta(1-\eta)\tilde{\Gamma}e^{-|t-t'|} \quad (17)$$

Following the method developed in Ref. [30], one can perform a homodyne detection of the outgoing field fluctuations with a temporally matched local oscillator,  $\mathcal{E}_{LO}(t) \sim e^{-\tilde{\Gamma}t}$  [32]. The measured quantity is then  $\Delta x_m^2 = 1 + \eta(n_T^i - n_T^f) + \eta(1-\eta)n_T^i$ , where  $n_T^{i,f}$  are the initial and final thermal excitation numbers. When the cooling is optimal ( $\eta \sim 1$ ), one indeed measures the change in temperature of the mirror:  $\Delta x_m^2 \simeq 1 + n_T^i - n_T^f$ .

We have presented a general theory for the self-cooling of a mechanical oscillator to the ground state, which allows for deriving analytical results for the mirror final temperature and provides a simple interpretation of the cavity mediated self-cooling process. When the cavity is suitably detuned the initial thermal noise of the mirror vibrational mode is essentially transferred to the cavity field mode. This noise exchange reflects in the field leaking out of the cavity, providing useful information on the mirror temperature.

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  - [32] This corresponds to measuring the fluctuations of  $a_m = \int dt \mathcal{E}_{LO}(t)a_{\text{out}}(t)$ .